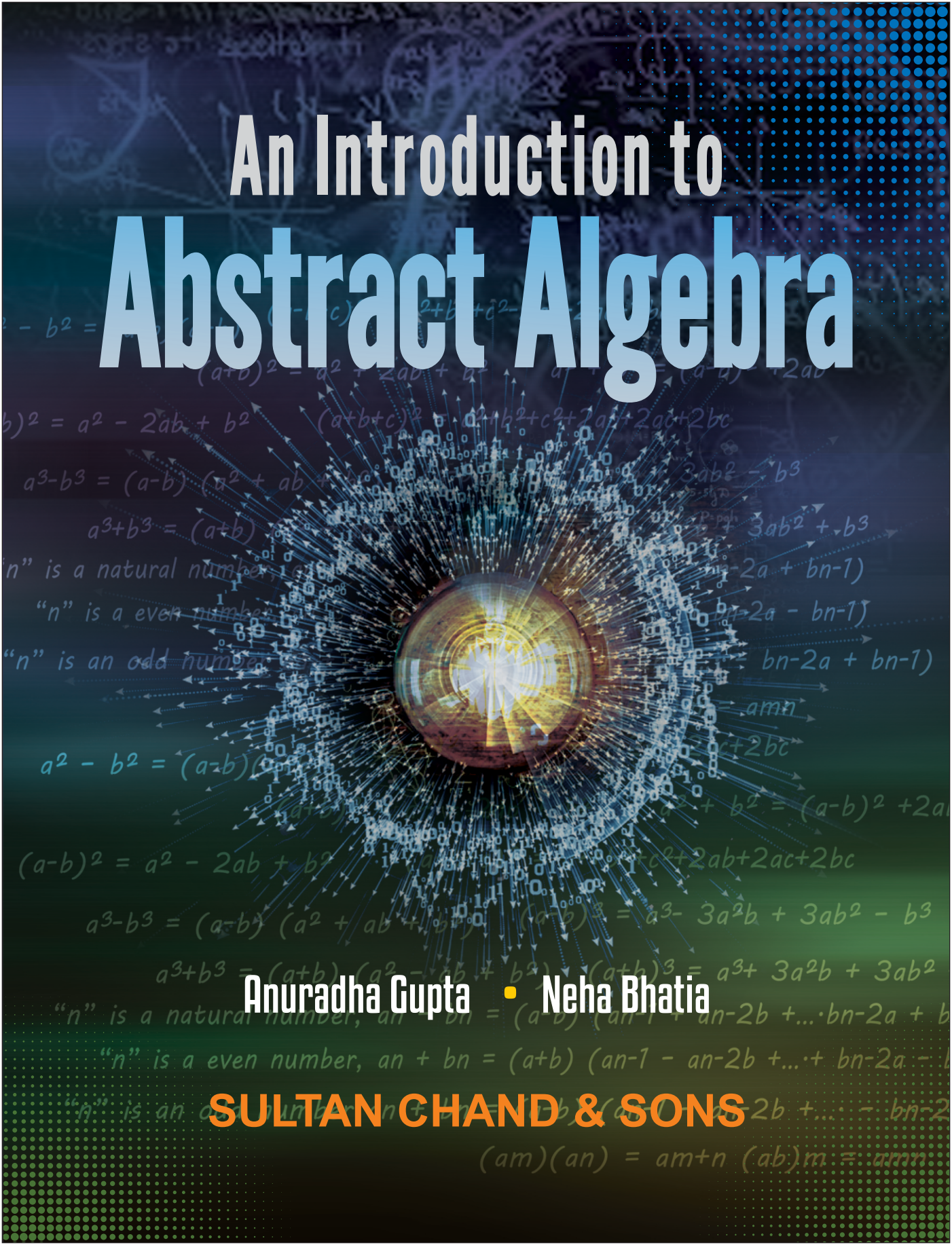
An Introduction to
Abstract Algebra



Anuradha Gupta • Neha Bhatia

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An Introduction to Abstract Algebra

To

My Daughters

Aishna & Dishita

Dr. Anuradha Gupta

To

My Parents

Mr. Vinod Bhatia

&

Mrs. Usha Bhatia

Dr. Neha Bhatia

An Introduction to Abstract Algebra

Dr. Anuradha Gupta
(Former Officiating Principal)
Associate Professor
Delhi College of Arts and Commerce
University of Delhi

Dr. Neha Bhatia
Assistant Professor
Daulat Ram College
University of Delhi

SULTAN CHAND & SONS
23, Daryaganj, New Delhi-110002
Tel.: 23266105, 23277843, 23247051, 23243183, 23281876
E-mail: sultanchand74@yahoo.com; info@sultanchandandsons.com
Website: www.sultanchandandsons.com
Fax: 011-2326-6357

SULTAN CHAND & SONS
23, Daryaganj, New Delhi-110002
Phones : 011-23281876, 23266105, 23277843 (*Showroom & Shop*)
011-40234454, 23247051 (*Office*)
E-mail : sultanchand74@yahoo.com; info@sultanchandandsons.com
Fax : 011-23266357; Website : www.sultanchandandsons.com

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Preface

Mathematics is the abstract key which turns the lock of the physical universe, a famous quote by a great physicist John Polkinghorne encourages us to explore the world of Abstract Algebra. Abstract is basically existing in thought or as an idea but not having a physical or concrete existence. Algebra is the branch of Mathematics which is the study of mathematical symbols. It began with computation similar to those of arithmetic where numbers are represented by the letters. It starts with the solving of the equations. Then more general questions, such as "does an equation have a solution?", "how many solutions does an equation have?", "what can be said about the nature of the solutions?" are considered. These questions led extending algebra to non-numerical objects, such as permutations, vectors, matrices, and polynomials. The structural properties of these non-numerical objects were then abstracted into algebraic structures such as groups, rings, and fields.

The term abstract algebra was coined in the early 20th century to distinguish this area of study from the other parts of algebra. Abstract algebra is a broad field of mathematics focused on algebraic structures such as groups, rings, vector spaces and fields.

Abstract Algebra is the advance form of algebra that deals with different algebraic structures rather than the usual number system. Why we should study Abstract Algebra? We should study abstract algebra because it provides another way to look at the same problem of classical algebra and it provides a different insights to the same problem.

This book has been designed for the students who will be entering the world of Abstract Algebra for the first time. Our aim is to provide a comfortable platform for the students in the form of this book where the learning of abstraction of algebra is easy.

During the high school, students are introduced to elementary algebra where they learn how to solve the real world problems with the help of the symbols and reduce the problems to equations which can be further solved to give the numerical value. This methodology of converting complicated problems into symbols or letters forms the basis of all problems in Mathematics. But the algebra is even beyond this technique.

Abstract form of Mathematics is different from other sciences. While in sciences such as chemistry, physics and zoology, a laboratory scientists perform test and experiments to analyze and discover new results and verify theories. Mathematicians use mathematical theory, logical arguments, computational techniques, algorithms, and the latest computer technology to solve economic, scientific, and business related problems. In studying abstract mathematics, we take an axiomatic approach, giving arguments in the form of the mathematical proof. We study fundamental algebraic structures, namely groups, rings, vector spaces and fields and maps between these structures.

Algebra has evolved slowly and gradually over the last few decades of the 20th century. Its growth has been associated with the development of other branches of mathematics, and it has been influenced by philosophical ideas and logic.

Our primary goal is to help the students in learning the beauty of the subject. The well explained proof and the examples will allow the students to connect with the outer world and to explore the

applications emerging out of the theory of Algebra. This book enable the students to learn and understand algebra as much as possible. The textbook covers all the topics related to the subject and has been designed to teach the principles and theory of abstract algebra to the students and the readers in rigorous manner. The strength of the textbook lies in the wide range of examples and exercises with many practical applications.

This text is intended for an undergraduate courses in abstract algebra. These courses have covered the theoretical and practical aspects of groups, rings, and fields. The content of the book is as per the Choice Based Curriculum System (CBCS) of various universities in India, This book intend to cater the need of third and fourth semester students of University of Delhi.

Anuradha Gupta
Neha Bhatia

Syllabus

BMH306: GROUP THEORY-I

Unit 1 : Groups and its Elementary Properties **(Lectures: 10)**

Symmetries of a square, The Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), Elementary properties of groups.

Unit 2: Subgroups and Cyclic Groups **(Lectures: 15)**

Subgroups and examples of subgroups, Centralizer, Normalizers, Center of a group, Product of two subgroups, Properties of cyclic groups, Classification of subgroups of cyclic groups.

Unit 3 : Permutation Groups and Lagrange's Theorem **(Lecturer: 25)**

Cycle notation for permutations, Properties of permutations, Even and odd permutations, alternating groups; Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem; Normal subgroups, factor groups, Cauchy's theorem for finite abelian groups.

Unit 4: Group Homomorphisms **(Lectures: 20)**

Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem, Properties of isomorphisms, First, Second and Third isomorphism theorems for groups.

BMH410: RING THEORY AND LINEAR ALGEBRA-I

Unit 1 : Introduction of Rings **(Lectures: 20)**

Definition and examples of rings, Properties of rings, Subrings, Integral domains and fields, characteristic of a ring, Ideals, Ideal generated by a subset of a ring, Factor rings, Operations on ideals, Prime and maximal ideals.

Unit 2: Ring Homomorphisms **(Lectures: 10)**

Ring homomorphisms, Properties of ring homomorphisms, First, Second and Third Isomorphism theorems for rings, The Field of quotients.

Unit 3: Introduction of Vector Spaces **(Lectures: 20)**

Vector spaces, Subspaces, Algebra of subspaces, Quotient spaces, Linear combination of vectors, Linear span, Linear independence, Basis and dimension, Dimension of subspaces.

Unit 4: Linear Transformations **(Lectures: 20)**

Linear transformations, Null space, Range, Rank and nullity of a linear transformation, Matrix representation of a linear transformation, Algebra of linear transformations, Isomorphisms, Isomorphism theorems, Invertibility and the change of coordinate matrix.

Contents

List of Symbols

$\mathbb{N}$	set of natural numbers
$\mathbb{Z}$	set of integers
$\mathbb{Q}$	set of rational numbers
$\mathbb{R}$	set of real numbers
$\mathbb{C}$	set of complex numbers
$\emptyset$	empty set
ϕ	phi
$\Rightarrow$	implies
$\Leftrightarrow$	if and only if or iff
$\exists$	there exist
$\forall$	for all
$\in$	is an element of
$\notin$	is not an element of
$\subseteq$	subset of
$P(A)$	power set of A
$A \times B$	Cartesian product of A and B
$f: A \to B$	f is a function from A to B
$\cup_{i \in I} X_i$	Union of the sets X_i
$\cap_{i \in I} X_i$	Intersection of the sets X_i
$ran\, f$	range of the function f
$Ker\, f$	kernel of the homomorphism f
I_A	identity function on A
A^c	complement of A
det	determinant
δ_{ij}	Kronecker delta
$a \mid b$	a divides b
$a \nmid b$	a does not divide b
(a, b)	greatest common divisor of a and b
$a \equiv b \pmod{n}$	a is congruent to b modulo n
$\sim$	is equivalent to
$\lvert X \rvert$	$\begin{cases} \text{cardinality of set } X, \\ \text{order of the group } X, \\ \text{determinant of the matrix } X \end{cases}$

$\|a\|$	order of the element a
S_n	symmetric group of n elements
A_n	alternating group of n elements
$\mathbb{Z}_n$	integers modulo n
$Z(G)$	center of a group G
aH, Ha	left and right cosets of H
$[G : H]$	index of a subgroup H in a group G
HK	$\{ab : a \in H, b \in K\}$
$H < G$	H is a proper subgroup of G
$N \triangleleft G$	N is a normal subgroup of G
G/N	factor group or quotient group of G by N
char R	characteristic of the ring R
α	alpha
β	beta
γ	gamma
I_n	$n \times n$ identity matrix
A^t	transpose of the matrix A
A^{-1}	inverse of the matrix A
dim V	dimension of the vector space V
$r(T)$	rank of linear transformation T
$\eta(T)$	nullity of linear transformation T

About the Book

This book on **Abstract Algebra** is intended for one or two semesters of B.Sc. (Hons.) and B.A. (Prog.) of University of Delhi and other Universities of India. The book is written in simple language to make the students understand various topics in Abstract Algebra in an easier way. The examples and exercises of the book are meticulously crafted and honed to meet the need of the students who are keen to know about Abstract Algebra. Starting from Set Theory and covering the topics on Groups, Rings and Vector Spaces, the book provides the students a deep study of Abstract Algebra. The book 'Abstract Algebra' combines the theory, examples with exercises on the concepts related to the topics in Abstract Algebra.

Salient Features of the Book

- The book has been designed as per the latest CBCS syllabus.
- It provides a clear understanding of the subject "Abstract Algebra".
- The language of the book is comprehensible and easy to understand which can be read with minimum guidance.
- Chapters of the book are organized in a reader-friendly manner.
- It consists of solved illustrations and practice questions after every section of the chapter for better understanding and more clarity of the concept.
- It consists of important Points of Observations.
- Recent question papers of various universities of India are included at the end of the book.

Dr Anuradha Gupta is teaching in the Department of Mathematics, Delhi College of Arts & Commerce, University of Delhi. She has a rich experience of 31 years of teaching undergraduate and post graduate students of University of Delhi. She is actively involved in research in the field of Operator theory. She has sixty five research papers published in National and International Journals of repute. Ten students have completed Ph.D. under her supervision from University of Delhi. She has also presented her research work in International Conferences in India and abroad. She has written books on Complex Analysis, Multivariable Calculus and Business Mathematics for undergraduate and post graduate students.

Dr. Neha Bhatia is an Assistant Professor at the Department of Mathematics, Daulat Ram College, University of Delhi. She has a teaching experience of more than 8 years and has taught a variety of subjects from Algebra, Differential Equations to Real Analysis at undergraduate level. She did her doctorate in 2015 in Theory of Operators from the Department of Mathematics, University of Delhi. She has various publications in reputed National and International Journals. She has also participated and presented her research work at National and International level and is fully committed to the welfare of the students fraternity.

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Phones: 23243183, 23247051, 23277843, 23281876, 23266105

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