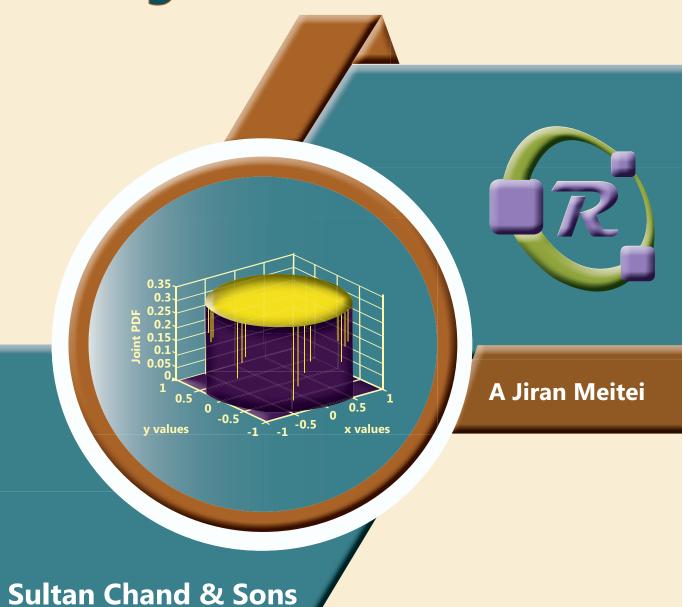
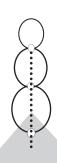
Probability Distributions AND Queueing Theory Using RAND Octave





PROBABILITY DISTRIBUTIONS AND

QUEUEING THEORY
USING R AND OCTAVE



to
My Family and Teachers

PROBABILITY DISTRIBUTIONS AND QUEUEING THEORY USING R AND OCTAVE

DR. A. JIRAN MEITEI

Associate Professor
Department of Mathematics
Maharaja Agrasen College
University of Delhi, Delhi



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INDIAN STATISTIAL INSTITUTE

(DELHI CENTRE)

Gram: STATISTICA, NEW DELHI

Phone: +91-11-41493958 Fax: +91-11-41493981 Mobile: +91-9999907648

Web : www.isis.ac.in/~shanta e-mail : shanta@isid.ac.in

head smu@isid.ac.in



Shanta Laishram
Professor of Mathematics &
Head, Stat Math Unit (SMUD)
Indian Statistical Institute
7 SJS Sansanwal Marg
New Delhi 110016 INDIA

Foreword

I am delighted to introduce the book titled *Probability Distributions and Queueing Theory Using R* and Octave authored by Dr. A. Jiran Meitei. This book is an invaluable resource designed specifically for students pursuing disciplines such as statistics, mathematics, operational research, economics, commerce, and related fields at both undergraduate and postgraduate levels. The primary objective of the book is to present the complex subject matter in a simple and user-friendly manner, ensuring that students can easily grasp the concepts and apply them effectively. The book places significant emphasis on the practical use of open-source software tools, R and Octave, which are readily available for download. It also incorporates the use of MS Excel, including R programs for generating statistical tables, enabling students to seamlessly work with various probability distributions.

Comprising nine comprehensive chapters, the book covers a wide range of topics related to probability distributions and queueing theory. Two dedicated chapters delve into stochastic processes and queueing theory, providing in-depth discussions and valuable insights. Each chapter focuses on specific topics and includes appropriate examples to facilitate better understanding. Furthermore, exercises at the end of each chapter allow students to practice solving a variety of problems, reinforcing their comprehension and analytical skills.

Throughout the book, the author demonstrates the use of different R packages, providing students with a comprehensive understanding of these tools for statistical analysis and problem-solving. In summary, *Probability Distributions and Queueing Theory Using R and Octave* is a comprehensive and accessible guide that aims to equip students with a strong foundation in probability distributions and queueing theory. By emphasizing the practical application of R, Octave, and MS Excel, this book ensures that students can confidently apply their knowledge in real-world scenarios. Dr. Meitei's expertise and dedication shine through the content, making this book an invaluable resource for students and practitioners alike.

July 24, 2023 Shanta Laishram

P Preface

The book titled 'Probability Distributions and Queueing Theory Using R and Octave' is authored by Dr. A. Jiran Meitei, and aims to cater to students pursuing disciplines such as statistics, mathematics, operational research (OR), economics, commerce, and related fields at both undergraduate and postgraduate levels. The author's primary goal is to present the content in a simple and user-friendly manner.

The book extensively focuses on demonstrating the practical utilization of R and Octave, which are open-source software tools readily available for download, for working with various probability distributions. Additionally, the book incorporates the use of MS Excel for working with these distributions, including R programs for generating statistical tables.

Comprising nine chapters, the book covers a wide range of topics related to probability distributions and queueing theory. Two chapters are dedicated specifically to stochastic processes and queueing theory. Each chapter covers specific topics in detail and provides appropriate examples to facilitate better understanding. Moreover, exercises are included at the end of every chapter to allow students to practice solving a variety of problems.

Chapter 1 serves as an introduction, providing an overview of different types of random variables and their associated probabilities. Octave programs for plotting the Probability Mass Function (PMF), Joint PMF, and Probability Density Function (PDF) are provided to aid readers. **Chapter 2** delves into expectations and their properties, Moment Generating Function (MGF), cumulants, and other aspects related to random variables.

Chapter 3 focuses on discrete probability distributions, including Binomial, Poisson, Geometric, Negative Binomial, and Hypergeometric distributions. It provides detailed discussions on the properties and interrelationships of these distributions. Chapters 4 and 5 cover important univariate continuous probability distributions. Chapter 6 explains significant sampling distributions, including their properties and interrelationships. Chapter 7 presents the fitting of various probability distributions, illustrating the use of the 'fitdistribus' package in R for fitting univariate distributions. The last two chapters, Chapters 8 and 9, are dedicated to stochastic processes and queueing theory. The book also explains the functioning of the 'markovchain' package in R and Jensen MS Excel add-ins.

Overall, the book aims to provide students with a comprehensive understanding of probability distributions and queueing theory, emphasizing the practical application of R, Octave, and MS Excel in statistical analysis and problem-solving.

viii Preface

In expressing gratitude, the author acknowledges the support received from his family, friends, teachers, and colleagues, without whom completing this book would not have been possible. The author also acknowledges the contribution of his students, specifically **Mr. Ashish Singh**, who provided corrections and feedback on the chapters. Furthermore, the author extends appreciation to the teachers who imparted knowledge during his college days and acknowledges the help received from colleagues in developing the content of the book.

Finally, the author extends his gratitude to the esteemed publisher, **Sultan Chand & Sons**, for accepting to publish this work without delay.

A Jiran Meitei

Packages Used

R

Package Name Description

fastGraph Fast Drawing and Shading of Graphs of Statistical Distributions

fitdistrplus Help to Fit of a Parametric Distribution to Non-Censored or Censored Data

diagram Visualising Simple Graphs, Flowcharts and Webs
LaplacesDemon Complete Environment for Bayesian Inference
fMultivar Modeling of Multivariate Financial Return

markovchain Package for Easily Handling Discrete Markov Chains in R

MASS Support Functions and Datasets for Venables and Ripley's MASS

After installation, we need to load the library of the package before starting to use it, for example:

install.packages("fastGraph")

library(fastGraph)

Octave

Package Name Description

statistics A collection of functions for statistical analysis including the probability

distributions.

We need to run **pkg load statistics** in the command window before start using this package.

MS Excel

Package Name Description

queue.xla This is a Jensen Excel Add-ins, in the Queuing Add-in and computes steady-

state measures associated with Poisson queuing models, non-Markovian queues, and networks of queues. Both open and closed Markovian queues are modeled. The program also simulates multiple channel queues using two methods, a discrete next-event simulation, and an entity simulation. (https://

utw11041.utweb.utexas.edu/ORMM/excel/queue.html)

SP Snapshot of the Book

	Chapter	Pages	Examples	Exercise Questions
1.	Random Variables	1-49	30	22
2.	Mathematical Expectation	51-109	41	34
3.	Special Univariate Discrete Probability Distributions	111-184	32	62
4.	Special Univariate Continuous Probability Distributions-I	185-252	22	47
5.	Special Univariate Continuous Probability Distributions-II	253-360	30	54
6.	Sampling Distributions	361-429	12	37
7.	Fitting of Distributions	431-490	14	19
8.	Stochastic Processes and Basics of Queueing Theory	491-564	24	62
9.	Markovian Queues	565-620	7	29
	Total		212	366

Contents

1	Ran	dom Variables	1-49
	1.0	Random Variable	3
	1.1	Discrete Random Variable	4
	1.2	Continuous Random Variable	6
	1.3	Cumulative Distribution Function	9
		1.3.1 Properties of Distribution Function	10
	1.4	Joint Distributions of Random Variables	16
		1.4.1 Joint Probability Mass Function	16
		1.4.2 Joint Probability Density Function	22
		1.4.3 Joint Cumulative Distribution Function	29
	1.5	Convolution	30
	1.6	Transformation of Random Variables	34
		1.6.1 Transformation of One-Dimensional Random Variable	34
		1.6.2 Transformation of Two-Dimensional Random Variable	39
		Exercise 1	43
2	Mat	thematical Expectation	51-109
	2.1	Expectation of a Random Variable	53
		2.1.1 Properties of Expectation	53
	2.2	Moments	64
		2.2.1 Relationship Between Standard and Raw Moments	65
	2.3	Generating Functions	67
		2.3.1 Probability Generating Function	68
		2.3.2 Moment Generating Function	70
		2.3.3 Characteristic Function	74
		2.3.4 Cumulant Generating Function	74
	2.4	Measure of Skewness and Kurtosis Based on Moments	77
	2.5	Expectation Inequalities	82

xii	Contents	
2.6	Conditional Expectation	94
	2.6.1 Properties of Conditional Expectation	95
2.7	Moments of Two Random Variables	97
2.8	Moments of Linear Combinations of Random Variables	100
	Exercise 2	103
3 Spec	cial Univariate Discrete Probability Distributions	111-184
3.0	Introduction	113
3.1	Binomial Distribution	113
	3.1.1 Cumulative Distribution Function of Binomial Distribution	114
	3.1.2 Use of MS Excel, R and Octave for Binomial Distribution	115
	3.1.3 Mean and Variance of Binomial Distribution	118
	3.1.4 Moment Generating Function of Binomial Distribution	119
	3.1.5 Cumulant Generating Function of Binomial Distribution	120
	3.1.6 Recurrence Relation for Probability Mass Function of	120
	Binomial Distribution	120
	3.1.7 Recurrence Relation for Moments of Binomial Distribution	120
	3.1.8 R Code to Find the r th Moment of Binomial Distribution	122
	3.1.9 Measure of Skewness and Kurtosis of Binomial Distribution	122
	3.1.10 Distribution of the Sum of Independent Binomial Random Variab	
2.2	3.1.11 Mode of Binomial Distribution	126
3.2	Poisson Distribution	132
	3.2.1 Cumulative Distribution Function of Poisson Distribution	133
	3.2.2 Poisson Distribution as a Limiting Case of Binomial Distribution	133
	3.2.3 Use of MS Excel, R and Octave for the Poisson Distribution	134
	3.2.4 Mean and Variance of the Poisson Distribution	136
	3.2.5 Moment Generating Function of Poisson Distribution	137
	3.2.6 Cumulant Generating Function of Poisson Distribution	137
	3.2.7 Recurrence Relation for Moments of Poisson Distribution3.2.8 R Code for Moments of Poisson Distribution	138
	3.2.9 Measure of Skewness and Kurtosis of Poisson Distribution	138
		139
	3.2.10 Recurrence Relation for Probability Mass Function of Poisson Distribution	140
	3.2.11 Sum of Independent Poisson Random Variables	140
	3.2.12 Mode of the Poisson Distribution	142
3.3	Geometric Distribution	146
5.5	3.3.1 Cumulative Distribution Function of Geometric Distribution	147

Γ		
	Contents	xiii
	Contents	28.8.8.8
L		

		3.3.2	Use of MS Excel, R and Octave for Geometric Distribution	147
		3.3.3	Moment Generating Function of Geometric Distribution	149
		3.3.4	Cumulant Generating Function of Geometric Distribution	150
			Mean and Variance of the Geometric Distribution	150
		3.3.6	Mode of Geometric Distribution	151
		3.3.7	Markovian Property of Geometric Distribution	151
		3.3.8	Recurrence Relation for Moments of Geometric Distribution	152
		3.3.9	Measure of Skewness and Kurtosis of Geometric Distribution	153
		3.3.10	R Code for Moments of a Geometric Distribution	153
		3.3.11	Recurrence Relation for Probability of Geometric Distribution	154
	3.4	Negati	ve Binomial Distribution	155
		3.4.1	Use of MS Excel and R for Negative Binomial Distribution	157
		3.4.2	Mean and Variance of Negative Binomial Distribution	159
		3.4.3	Moment Generating Function of Negative Binomial Distribution	160
		3.4.4	Cumulant Generating Function of Negative Binomial Distribution	161
		3.4.5	Mean and Variance of Negative Binomial Distribution using	
			Moment Generating Function	161
		3.4.6	Deriving Mean and Variance of Negative Binomial Distribution	
			using Geometric Distribution	163
		3.4.7	Deriving Poisson Distribution as a Limiting Case of Negative	
			Binomial Distribution	163
			Recurrence Relation for Moments of Negative Binomial Distribution	
			Measure of Skewness and Kurtosis of Negative Binomial Distributio	n 166
		3.4.10	Recurrence Relation for Probability Mass Function of Negative	
			Binomial Distribution	167
		3.4.11	R Code to Find the Moments, Skewness and Kurtosis of	
	2.5		Negative Binomial Distribution	167
	3.5		geometric Distribution	170
			Mean and Variance of Hypergeometric Distribution	171
			Use of MS Excel, R and Octave for Hypergeometric Distribution	172
			R Code for Skewness and Kurtosis of Hypergeometric Distribution	175
		Exercis	se 3	177
4	Spe	cial Un	nivariate Continuous Probability Distributions-I	35-252
	4.1	Unifor	m Distribution	187
		4.1.1	Cumulative Distribution Function of Uniform Distribution	188
		4.1.2	Mean and Variance of Uniform Distribution	188
		4.1.3	Uniform Distribution using MS Excel, R, and Octave	189

xiv	Contents
XÍV	Contents

		4.1.4	Moment Generating Function of Uniform Distribution	191
		4.1.5	Moments of a Uniform Distribution	191
		4.1.6	Skewness and Kurtosis of Uniform Distribution	192
		4.1.7	R Code of Moments, Skewness and Kurtosis of Uniform Distribution	on 192
	4.2	Gaussi	an or Normal Distribution	196
		4.2.1	Cumulative Distribution Function of Normal Distribution	197
		4.2.2	Properties of a Normal Distribution	198
		4.2.3	Normal Distribution using MS Excel, R and Octave	209
		4.2.4	R Code for Generating Standard Normal Table	210
		4.2.5	Normal Distribution as a Limiting Case of Binomial Distribution	214
		4.2.6	Normal Distribution as a Limiting Case of Poisson Distribution	217
		4.2.7	Asymptotic Normality	230
		4.2.8	Central Limit Theorem	230
		4.2.9	Demonstration of CLT Using R	233
	4.3	Lognor	rmal Distribution	234
		4.3.1	Lognormal Distribution using MS Excel, R and Octave	236
		4.3.2	Moments, Skewness and Kurtosis of Lognormal Distribution	238
		4.3.3	Median and Mode of Lognormal Distribution	240
		4.3.4	R Code for Moments, Skewness, and Kurtosis of	
			Lognormal Distribution	241
			Product of Independent Lognormal Random Variables	
		4.3.5 Exercis	•	241 246
5	Sno	Exercis	se 4	246
5	Spe	Exercis	se 4	
5	Spe 5.1	Exercis	se 4	246 253-360
5	-	Exercise Expone	se 4 livariate Continuous Probability Distributions-II 2	246 253-360 255
5	-	cial Un Expone 5.1.1	se 4 nivariate Continuous Probability Distributions-II 2 ential Distribution	246
5	-	Expone 5.1.1 5.1.2 5.1.3	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution	246 253-360 255 256 257
5	-	Exercise 5.1.1 5.1.2 5.1.3 5.1.4	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution	246 253-360 255 256 257 257
5	-	Expone 5.1.1 5.1.2 5.1.3 5.1.4 5.1.5	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution Skewness and Kurtosis of Exponential Distribution	246 253-360 255 256 257 257 258 259
5	-	Expone 5.1.1 5.1.2 5.1.3 5.1.4 5.1.5 5.1.6	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution Skewness and Kurtosis of Exponential Distribution R Code for Moments of Exponential Distribution	246 253-360 255 256 257 257 258 259 259
5	-	Exercise 5.1.1 5.1.2 5.1.3 5.1.4 5.1.5 5.1.6 5.1.7	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution Skewness and Kurtosis of Exponential Distribution R Code for Moments of Exponential Distribution Markovian Property of Exponential Distribution	246 253-360 255 256 257 257 258 259 259
5	-	Expone 5.1.1 5.1.2 5.1.3 5.1.4 5.1.5 5.1.6 5.1.7 5.1.8	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution Skewness and Kurtosis of Exponential Distribution R Code for Moments of Exponential Distribution Markovian Property of Exponential Distribution Deriving Exponential Distribution from Poisson Process	246 253-360 255 256 257 258 259 259 260 260
5	5.1	Expone 5.1.1 5.1.2 5.1.3 5.1.4 5.1.5 5.1.6 5.1.7 5.1.8 5.1.9	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution Skewness and Kurtosis of Exponential Distribution R Code for Moments of Exponential Distribution Markovian Property of Exponential Distribution Deriving Exponential Distribution from Poisson Process Exponential Distribution using Octave	253-360 255 256 257 257 258 259 260 260 261
5	-	Exercise 5.1.1 5.1.2 5.1.3 5.1.4 5.1.5 5.1.6 5.1.7 5.1.8 5.1.9 Gamm	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution Skewness and Kurtosis of Exponential Distribution R Code for Moments of Exponential Distribution Markovian Property of Exponential Distribution Deriving Exponential Distribution from Poisson Process Exponential Distribution using Octave a Distribution	253-360 255 256 257 257 258 259 260 260 261 276
5	5.1	Expone 5.1.1 5.1.2 5.1.3 5.1.4 5.1.5 5.1.6 5.1.7 5.1.8 5.1.9 Gamm 5.2.1	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution Skewness and Kurtosis of Exponential Distribution R Code for Moments of Exponential Distribution Markovian Property of Exponential Distribution Deriving Exponential Distribution from Poisson Process Exponential Distribution using Octave a Distribution Cumulative Distribution Function of Gamma Distribution	246 2 53-360 255 256
5	5.1	Expone 5.1.1 5.1.2 5.1.3 5.1.4 5.1.5 5.1.6 5.1.7 5.1.8 5.1.9 Gamm 5.2.1 5.2.2	ential Distribution Mean and Variance of Exponential Distribution Moment Generating Function of Exponential Distribution Cumulant Generating Function of Exponential Distribution Moments of Exponential Distribution Skewness and Kurtosis of Exponential Distribution R Code for Moments of Exponential Distribution Markovian Property of Exponential Distribution Deriving Exponential Distribution from Poisson Process Exponential Distribution using Octave a Distribution	253-360 255 256 257 257 258 259 260 260 261 276

Contents

	5.2.4	Moment Generating Function of Gamma Distribution
	5.2.5	Cumulant Generating Function of Gamma Distribution
	5.2.6	Moments of Gamma Distribution
	5.2.7	Skewness and Kurtosis of Gamma Distribution
	5.2.8	Gamma Distribution from Poisson Process
	5.2.9	Gamma Distribution using Octave
5.3	Erlang	Distribution
5.4	Beta D	istribution
	5.4.1	Moments of Beta Distribution
	5.4.2	Moment Generating Function of Beta Distribution
	5.4.3	Mode of Beta Distribution
	5.4.4	The Beta Distribution of Second Kind
	5.4.5	Relationship between Beta Distribution of First and Second Kind
	5.4.6	Beta Distribution using Octave
	5.4.7	Relationship between Gamma and Beta Distribution
5.5	Weibul	ll Distribution
	5.5.1	Mean and Variance of Weibull Distribution
	5.5.2	Mode of Weibull Distribution
	5.5.3	Moment Generating Function of Weibull Distribution
	5.5.4	Weibull Distribution using Octave
5.6	Cauchy	y Distribution
	5.6.1	Cauchy Distribution using Octave
	5.6.2	R Code for Standard Cauchy Table
	5.6.3	Properties of Cauchy Distribution
5.7	Double	e-Exponential Distribution
	5.7.1	Mean and Variance of Laplace Distribution
	5.7.2	Moments of Laplace Distribution
		Skewness and Kurtosis of Laplace Distribution
	5.7.4	Moment Generating Function of Laplace Distribution
	5.7.5	Cumulant Generating Function of Laplace Distribution
	5.7.6	Laplace using Octave
	5.7.7	Some Important Properties of Laplace Dstribution
5.8	Logisti	c Distribution
	5.8.1	Mean and Variance of Logistic Distribution
	5.8.2	Moment Generating Function of Logistic Distribution
	5.8.3	Logistic Distribution using Octave
	5.8.4	Some Important Properties of Logistic Distribution
	Exercis	se 5

6	Sam	pling	Distributions	361-429
	6.0	Introdu	action	363
	6.1	Chi-Sq	uare Distribution	363
		6.1.1	Mean and Variance of Chi-Square Distribution	365
		6.1.2	Moments of the Chi-Square Distribution	366
		6.1.3	Moment Generating Function of the Chi-Square Distribution	367
		6.1.4	Cumulant Generating Function of Chi-Square Distribution	367
		6.1.5	Skewness and Kurtosis of Chi-Square Distribution	368
		6.1.6	Some Important Properties of Chi-Square Distribution	368
		6.1.7	Chi-Square Distribution using R	375
		6.1.8	Areas of Applications for the Chi-Square Statistic	376
		6.1.9	Code for Generating Chi-Square Table	377
	6.2	Studen	t t-Distribution	384
		6.2.1	Moments of t-Distribution	387
		6.2.2	Mean and Variance of t-Distribution	388
		6.2.3	Skewness and Kurtosis of the Distribution	390
		6.2.4	Relationship between t and other Distributions	390
		6.2.5	Assumptions and Areas of Applications for t-Distribution	393
		6.2.6	Use of R in t-Distribution	393
		6.2.7	R Code for Generating t-Table	395
	6.3	F-Dist	ribution	400
		6.3.1	Moments of F-Distribution	402
		6.3.2	Relationship Between F and Other Distributions	403
		6.3.3	F-Distribution using R	407
		6.3.4	Assumption and Areas of Applications of F-Distribution	409
		6.3.5	R Code for F-Table	409
		Exercis	se 6	424
7	Fitti	ing of	Distributions	431-490
	7.0	Introdu	action	433
	7.1	Steps I	nvolved in the Fitting of Distribution	433
		7.1.1	Choosing of the Candidate Distribution	433
		7.1.2	Estimation of Parameters	436
		7.1.3	Model Comparison using AIC, BIC, and Log-Likelihood	439

Contents			xvii
	7.1.4	Test for Goodness of Fit	439
7.		g of Discrete Distribution	440
	_	Fitting of Binomial Distribution	440
		Fitting of Poisson Distribution	447
		Fitting of Geometric Distribution	450
		Fitting of Negative Binomial Distribution	453
	7.2.5	Fitting of Hypergeometric Distribution	457
7.		g of Continuous Distribution	460
	7.3.1	Fitting of Normal Distribution	460
	7.3.2	Fitting of Exponential Distribution	464
	7.3.3	Fitting of Gamma Distribution	467
	7.3.4	Fitting of Weibull Distribution	470
7.	4 Introd	uction to Fitdistrplus Package for Fitting of Distribution using R	473
	Exerci	se 7	488
8 St	ochastic	Processes and Basics of Queueing Theory	491-564
8.	0 Stocha	astic Process	493
8.	1 Classi	fication Based on the Nature of the Parameter and State Space	493
8.	2 Classi	fication Based on Inter-dependence of the Random Variables	494
	8.2.1	Stationary Stochastic Processes	494
	8.2.2	Gaussian Process	498
	8.2.3	Random Walk	501
	8.2.4	The Wiener Process	505
	8.2.5	Martingales	509
	8.2.6	Markov Process	511
	8.2.7	Counting Process	512
	8.2.8	Poisson Process	513
8.	3 Discre	te Time Markov Chain	518
	8.3.1	Chapman-Kolmogorov Equations for DTMC	521
	8.3.2	Marginal Probabilities of a Markov Chain	523
	8.3.3	Classification of Chains and States of the Markov Chain	524
	8.3.4	Stationary and Limiting Distributions	532
	8.3.5	Calculation of Stationary Distributions	533
8.	4 Contin	nuous Time Markov Chain	536
	8.4.1	Chapman-Kolmogorov Equations of CTMC	539
	8.4.2	Infinitesimal Matrix of the CTMC	539

xviii		Contents	
		8.4.3 Classification of States	541
		8.4.4 Kolmogorov Differential Equations	541
	8.5	Birth and Death Processes	544
	8.6	Introduction to Markov Chain Package in R	545
	8.7	Queueing System	550
		8.7.1 Characteristics of the System	551
		8.7.2 Notation of System	554
		8.7.3 Measures of Performance of a Queueing System	555
		8.7.4 The Traffic Intensity of the System	556
		8.7.5 Little's Formula	556
		8.7.6 Transient and Steady-State Behavior	557
		8.7.7 Deterministic Queueing Model	558
		Exercise 8	558
9	Mar	kovian Queues	565-620
	9.0	Introduction	567
	9.1	$M/M/1$: $\infty/FCFS$	567
		9.1.1 State Independent	568
		9.1.2 State Dependent	575
	9.2	Multiple Servers Queueing System	585
	9.3	Finite Capacity Queue	586
	9.4	Finite Source System	587
	9.5	Queueing using MS Excel	594
	9.6	Transient Solution of the Markovian System	597
		9.6.1 The Expressions which are in the Form of the Infinite Sum of	
		Bessel Functions	597
		9.6.2 Other Expressions in the Form of Infinite Sum	598
		9.6.3 Expressions which Avoid Infinite Sum	599
		9.6.4 Expressions which are Put in the Integral Form of Some Kind	600
		9.6.5 Transient Solution of $M/M/1$: $\infty/FCFS$	601
		9.6.6 Busy Period Distribution	606
	9.7	Simulation and Queueing	608
	9.8	Types of Queueing Models	614
		Exercise 9	616
R	Refer	ences	621-623
I	ndex		625-633

About the Book

This book enables learners of Statistics, Mathematics, Operational Research, Economics, Commerce, and related disciplines to understand univariate probability distributions using R and Octave. The explanations and presentations in the book are simple and user-friendly. The book also focuses on Stochastic Process and Queueing Theory. Ample examples and exercise questions supplement the discussions in each chapter. All the statistical tables in the book are generated using the R (R codes are included for readers). The extensive use of MS Excel for working with distributions is another added feature.

Salient Features

- The book covers all the major univariate probability distributions and the basics of stochastic processes along with the Markovian queueing systems.
- O The New Educational Policy framework promotes the use of open-source softwares. R and Octave are two powerful open-source softwares which are freely accessible on the internet.
- O Each distribution is explained using R, Octave and MS Excel.
- All chapters are supplemented with ample examples.
- The book's content will be helpful to students of statistics, mathematics, operational research, business economics, and management.

About the Author

Dr A Jiran Meitei is an Associate Professor in the Department of Mathematics at Maharaja Agrasen College, University of Delhi. Jiran has a postgraduate degree in Operational Research and a doctorate in Statistics. He has taught various disciplines of Mathematical Sciences for more than 21 years to, undergraduate and postgraduate students. He has several research publications in various national and international journals. His areas of interest include probability, stochastic processes, queueing theory, linear & non-linear programming, machine learning, and artificial intelligence.





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Phones (S): 011-23281876, 23266105, 41625022

(0): 011-23247051, 40234454 Email : sultanchand74@yahoo.com

info@sultanchandandsons.com



